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ON THE EFFECT OF SUBSONIC TRAILING EDGES ON DAMPING IN ROLL AND PITCH OF THIN SWEPTBACK WINGS IN A SUPERSONIC STREAM

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ON THE EFFECT OF SUBSONIC TRAILING EDGES ON DAMPING IN ROLL AND PITCH OF THIN SWEPTBACK WINGS IN A SUPERSONIC STREAM

By Herbert S. Ribner

SUMMARY

The principal effect of subsonic trailing edges on the damping in roll and pitch of thin sweptback wings in a supersonic stream is evaluated with the aid of some conical and quasi-conical flows previously derived. This effect is expressed in the form of approximate correction terms to be added to the corresponding expressions that are obtained when the trailing-edge disturbance is ignored. The results are limited to those plan forms and Mach numbers for which the trailing-edge disturbance does not extend beyond the leading edge and for which the area of mutual interference between tips and trailing edge is not large. Practical applicability is subject to the limitations of linearized potential theory.

INTRODUCTION

At supersonic speeds the flow over a sweptback wing is closely related to the flow over a triangular or delta wing (fig. 1(a)) with the same leading-edge sweep angle. Thus, in figure 1(b), the flow characteristic of the delta persists out to the tip Mach cones. Within the tip Mach cones the flow is altered. (See reference 1.) At lower supersonic speeds (fig. 1(c)), the conical Mach wave from the apex of the trailing edge encloses a portion of the trailing edge. Here also the basic delta flow is altered. (At these speeds the component stream velocity normal to the trailing edges is subsonic and the edges are termed "subsonic.")

The altered flow in the tip and trailing-edge regions may be considered to result from superposing on the basic delta-wing flow certain necessary additional flows. These additional flows have the function of canceling the delta-wing lift outside the wing tips and behind the trailing edge. These cancellation flows yield the disturbance pressures in the tip regions and, if the edge is subsonic, in the trailing-edge region.

According to figure 1(c), the tip and trailing-edge disturbance regions overlap. In the cross-hatched upper portion of the overlap, for each tip, the trailing-edge disturbance and the tip disturbance merely superpose. Secondary disturbances in the black lower portion of the overlap are caused by mutual interference between the tip and the trailing edge. Under fairly general circumstances, these secondary disturbances have been found (reference 1) to contribute a relatively unimportant amount to the integrated lift and moment. Accordingly, the secondary disturbances are neglected herein and the tip and trailing-edge disturbances are assumed to be independent and to superpose within the entire region of overlap. The problem of the tip disturbance has received a rigorous treatment in references 1 and 2 and a simplified approximate treatment in reference 3. The discussion herein is limited to the trailing-edge disturbance.

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The complete flow necessary to cancel the lift behind a subsonic trailing edge is difficult to evaluate. It is shown in reference 1, however, for the case of angle of attack, that the major part of the disturbance pressure in the subsonic-trailing-edge region is obtained with only partial cancellation. (The complete cancellation is, however, carried out to close approximation in reference 1.) This partial cancellation is afforded by a certain conical flow (designated flow I) in the manner shown in figure 2(a). The flat portion of the load distribution in section A-A represents a uniform negative lift in the triangular region behind the trailing edge; its magnitude is chosen to equal the value of the basic delta-wing flow at the midspan. The outer portions constitute the sought-for pressure disturbance on the wing in the trailing-edge region.

Similar concepts can be employed to evaluate in simple fashion the principal portion of the subsonic-trailing-edge pressure disturbance in the case of rolling or pitching. For rolling motion the basic lift to be canceled is antisymmetric. The appropriate partial cancellation is afforded by a certain flow III in the manner shown in figure 2(b). For pitching motion the basic lift to be canceled increases linearly downstream. The appropriate partial cancellation is afforded by superposition of a constant negative lift (flow I) and another lift (flow IV) that increases in proportion to the distance downstream of the apex of the trailing edge (fig. 2(c)).

The detailed derivation of these several cancellation flows or "wake corrections" and others is carried through in reference 4. The present report is concerned with the application of some of these results to the determination of damping in roll and pitch for thin

swept wings with subsonic trailing edges. These applications take the form of correction terms to be added to the corresponding expressions that are obtained when the trailing-edge disturbance is ignored.

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The effect of subsonic trailing edges on the damping in roll of sweptback wings, which is part of the present subject, has also been treated recently in reference 2. The treatment therein achieves nearly exact cancellation of the lift behind the trailing edge by the superposition of infinitely many infinitesimal flows. The procedure is a development of the approach used in reference 1. The present treatment differs in employing approximate cancellation by superposition of a single flow. This simplification permits giving the over-all result compactly in closed form.

The results presented herein are applicable for those supersonic speeds for which the trailing-edge disturbance does not extend beyond the leading edge. At the lower Mach numbers for which the leading edge is enveloped further correction is required. This low Mach number problem is studied for the case of angle of attack in reference 5.

CORRECTION FOR DAMPING IN ROLL

For a sweptback wing with subsonic leading edges rolling with angular velocity p, the basic delta-wing lift distribution to be canceled behind the trailing edge is (equation (26), reference 6)

$$C_{p,\Delta} = \frac{2(x+c_r)ypm^2 I(m)}{\beta V \sqrt{(x+c_r)^2 m^2 - \beta^2 y^2}} \tag{1}$$

where

$$I(m) = \frac{2(1-m^2)}{(2-m^2)E'(m) - m^2 K'(m)}$$

and x is measured from the apex of the trailing edge. (All symbols are defined in appendix A.) This load distribution has the antisymmetric character shown in figure 2(b). For the inexact cancellation employed herein, the distribution is approximated by its tangent at $y=0$:

$$C_{p,\Delta} \approx \left(\frac{\partial C_{p,\Delta}}{\partial y}\right)_{y=0} y$$

$$\approx \frac{2pmI(m)}{\beta V} y \tag{2}$$

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(The limitations on the use of this approximation are considered in the DISCUSSION.)

The load-distribution equation (2) can be canceled by flow III of reference 4, which can be written as

$$C_p = Cy \tag{3}$$

where

$$C = \text{constant}$$

in the region $0 \leq |\sigma| \leq n$ behind the trailing edge of the wing. Thus the choice

$$C = -\frac{2pmI(m)}{\beta V} \tag{4}$$

achieves the desired cancellation.

Cancellation flow III includes also a lift disturbance ahead of the trailing edge in the region $n \leq |\sigma| \leq 1$. Reference 4 gives for this disturbance, in the present notation,

$$C_p = Cx \left\{\frac{\sigma\left[E(\phi,k)-n^2F(\phi,k)\right] - \sqrt{(1-\sigma^2)(\sigma^2-n^2)}}{\beta\left[E(k)-n^2K(k)\right]}\right\} \tag{5}$$

where

C = same constant as in equation (3)

$$\phi = \sin^{-1}\sqrt{\frac{1-\sigma^2}{1-n^2}}$$

$k = \sqrt{1-n^2}$

$\sigma = \beta y/x$

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The subsonic-trailing-edge correction to the damping in roll results from integrations to obtain the contribution of equation (5) to the rolling moment of the wing.

The correction is evaluated as follows: Equation (5) is written in the form

$$C_p = Cxf(\sigma)$$

Then the incremental rolling moment on an elementary triangular region (hatched area, fig. 3) is given by

$$d\left(\frac{\Delta L'}{\frac{1}{2}\rho V^2}\right) = -\int_{x=0}^{\frac{\beta b}{2\sigma}} yCxf(\sigma)\ x\ dx\ \frac{d\sigma}{\beta}$$

$$= -\frac{C\beta^2 b^4 f(\sigma)\ d\sigma}{64\sigma^3}$$

The incremental rolling moment experienced by the finite triangular region $n \leq \sigma \leq 1$ of figure 3 is given by

$$\frac{\Delta L'}{\frac{1}{2}\rho V^2} = -\frac{C\beta^2 b^4}{64}\int_n^1 \frac{f(\sigma)}{\sigma^3}\ d\sigma \tag{6}$$

The function $f(\sigma)$ is the expression in braces in equation (5). Thus the integrand in equation (6) involves elliptic integrals and related functions. The integration may be effected without difficulty with the aid of the elliptic-function substitutions of appendix B. The result is

$$\frac{\Delta L'}{\frac{1}{2}\rho V^2} = -\frac{C\beta b^4}{64n}\left[1 - \frac{\pi}{4}\frac{1-n^2}{E(k)-n^2K(k)}\right] \tag{7}$$

The following standard definitions are convenient:

$$E(k) = E'(n)$$

$$K(k) = K'(n)$$

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With these substitutions, equation (7) becomes

$$\frac{\Delta L'}{\frac{1}{2}\rho V^2} = -\frac{C\beta b^4}{64n}\left[1 - \frac{\pi}{4}\frac{1-n^2}{E'(n)-n^2K'(n)}\right] \tag{8}$$

The value of C given by equation (4) is to be substituted back in equation (8). Then the expression may be converted to coefficient form by division by Sb and by $pb/2V$. Multiplication by a factor of 2 accounts for both wing panels on the complete wing. The final result is (with both sides multiplied by β)

$$\Delta(\beta C_{l_p}) = \frac{A\beta m I(m)}{8n}\left[1 - \frac{\pi}{4}\frac{1-n^2}{E'(n)-n^2K'(n)}\right] \tag{9}$$

in which the leading- and trailing-edge sweep parameters m and n, respectively, are related by

$$\frac{1}{n} = \frac{1}{m} - \frac{4}{A\beta}\frac{1-\lambda}{1+\lambda} \tag{10}$$

where A is the aspect ratio and λ the taper ratio of the wing.

Equation (9) gives a correction for the damping-in-roll derivative C_{l_p} when the trailing edge of the wing is subsonic. This correction is algebraically additive to the value of βC_{l_p} computed, as in reference 3 (in effect), by integration of the basic delta-wing lift distribution together with the primary tip correction.

Applicability of equation (9) is limited to those plan forms and supersonic speeds for which the trailing-edge disturbance does not envelope the leading edge of the wing. This limitation is a consequence of the shape of the region of integration assumed in figure 3. The limitation may be expressed by

$$1 \leq \frac{1}{n} \leq 1 + \frac{4}{A\beta}\frac{\lambda}{1+\lambda} \tag{11a}$$

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or alternatively by

$$1 + \frac{4}{A\beta}\frac{1-\lambda}{1+\lambda} \leq \frac{1}{m} \leq 1 + \frac{4}{A\beta(1+\lambda)} \tag{11b}$$

CORRECTION FOR DAMPING IN PITCH

In the case of a sweptback wing with subsonic leading edge pitching about its apex, the basic delta-wing lift to be canceled behind the trailing edge is (equation (29), reference 6):

$$C_{p,\Delta} = \frac{4qG(m)\left[2(x+c_r)^2m^2-\beta^2y^2\right]}{\beta V\sqrt{(x+c_r)^2m^2-\beta^2y^2}} \tag{12}$$

where

$$G(m) = \frac{1-m^2}{(1-2m^2)E'(m)+m^2K'(m)}$$

and x is measured from the apex of the trailing edge. Now the variation of $C_{p,\Delta}$ with y is very gradual for a certain range of y; for the approximate cancellation employed herein, this variation is neglected. Thus the lift canceled is given by equation (12) with y therein set equal to zero.

$$C_{p,\Delta} = \frac{8c_rqmG(m)}{\beta V} + \frac{8qmG(m)}{\beta V}x \tag{13}$$

(The replacement of the exact curve (equation (12)) by its tangent (equation (13)) should give unusually accurate results because the first three derivatives of the function with respect to y vanish at $y = 0$. The limitations on the use of this approximation are considered in the DISCUSSION.) By suitable choice of constants, the first term can be canceled by flow I of reference 3 and the second term by flow IV of reference 4, which are of the form

$$\left.\begin{aligned} \text{flow I:}\quad C_p &= C_I \\ \text{flow IV:}\quad C &= C_{IV}x \end{aligned}\right\} \quad 0 \leq \sigma \leq n \tag{14}$$

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Thus the choice

$$\left.\begin{aligned} C_I &= -\frac{8c_r qmG(m)}{\beta V} \\ C_{IV} &= -\frac{8qmG(m)}{\beta V} \end{aligned}\right\} \tag{15}$$

achieves the desired cancellation.

Equations (14) refer to the portions of the cancellation flows behind the wing trailing edge (that is, $0 \leq |\sigma| \leq n$). Each of these flows also includes a lift disturbance ahead of the trailing edge in the region $n \leq |\sigma| \leq 1$. Reference 4 gives for these disturbances

$$\left.\begin{aligned} \text{flow I:}\quad C_p &= C_I \frac{F(\phi,k)}{K(k)} \\ \text{flow IV:}\quad C_p &= C_{IV}x \left[\frac{F(\phi,k)-E(\phi,k)}{K(k)-E(k)}\right] \end{aligned}\right\} \quad n \leq |\sigma| \leq 1 \tag{16}$$

The subsonic-trailing-edge correction to the damping in pitch results from integrations to obtain the contribution of equations (16) to the pitching moment of the wing.

The pitching-moment correction referred to the apex of the wing is desired, but it is more convenient at first to refer the moments to the apex of the trailing edge (y-axis of fig. 3). This procedure entails obtaining both lifts and moments, inasmuch as the lift is involved in the transfer of the moment reference axis back to the wing apex. The first of equations (16) (flow I) is of the form $C_p = C_I f(\sigma)$. The lift and the moment imparted by flow I to the elementary hatched triangle of figure 3 are therefore

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$$d\left(\frac{\Delta_I L}{\frac{1}{2}\rho V^2}\right) = \int_0^{\frac{\beta b}{2\sigma}} C_I f(\sigma)\ x\ dx\ \frac{d\sigma}{\beta}$$

$$= \frac{C_I \beta b\ f(\sigma)\ d\sigma}{8\sigma^2}$$

$$d\left(\frac{\Delta_I M}{\frac{1}{2}\rho V^2}\right) = -\int_0^{\frac{\beta b}{2\sigma}} x C_I f(\sigma)\ x\ dx\ \frac{d\sigma}{\beta}$$

$$= -\frac{C_I \beta^2 b^3 f(\sigma)\ d\sigma}{24\sigma^3}$$

The lift and the moment experienced by the finite triangular area of figure 3 are obtained by integrating between the limits n and 1. The function $f(\sigma)$ therein is the coefficient of C_I in equations (16). The substitutions of appendix B are helpful in effecting the integrations. The results are

$$\frac{\Delta_I L}{\frac{1}{2}\rho V^2} = \frac{C_I \beta b^2}{8n}\left[1 - \frac{\pi/2}{K'(n)}\right] \tag{17}$$

$$\frac{\Delta_I M}{\frac{1}{2}\rho V^2} = -\frac{C_I \beta^2 b^3}{48n^2}\left[1 - \frac{E'(n)}{K'(n)}\right] \tag{18}$$

The second of equations (16) (flow IV) is of the form $C_p = C_{IV} x f(\sigma)$. The lift and the moment imparted by flow IV to the elementary hatched triangle of figure 3 are therefore

$$d\left(\frac{\Delta_{IV}L}{\frac{1}{2}\rho V^2}\right) = \int_0^{\frac{\beta b}{2\sigma}} C_{IV}xf(\sigma)\ x\ dx\ \frac{d\sigma}{\beta}$$

$$= \frac{C_{IV}\beta^2 b^3 f(\sigma)\ d\sigma}{24\sigma^3}$$

$$d\left(\frac{\Delta_{IV}M}{\frac{1}{2}\rho V^2}\right) = -\int_0^{\frac{\beta b}{2\sigma}} xC_{IV}xf(\sigma)\ x\ dx\ \frac{d\sigma}{\beta}$$

$$= \frac{-C_{IV}\beta^3 b^4 f(\sigma)\ d\sigma}{64\sigma^3}$$

The lift and the moment experienced by the finite triangular area of figure 3 are obtained, as before, by integrating between the limits n and 1. The function $f(\sigma)$ is the expression in brackets in the second of equations (16). The result of the integrations is

$$\frac{\Delta_{IV}L}{\frac{1}{2}\rho V^2} = \frac{C_{IV}\beta^2 b^3}{48n^2}\left[1 - \frac{E'(n)-n^2K'(n)}{K'(n)-E'(n)}\right] \tag{19}$$

$$\frac{\Delta_{IV}M}{\frac{1}{2}\rho V^2} = -\frac{C_{IV}\beta^3 b^4}{192n^3}\left[1 - \frac{\pi}{4}\frac{1-n^2}{K'(n)-E'(n)}\right] \tag{20}$$

All the data are now at hand for calculating the incremental pitching moment referred to the wing apex. First the incremental lift will be needed.

$$\frac{\Delta L}{\frac{1}{2}\rho V^2} = \frac{\Delta_I L}{\frac{1}{2}\rho V^2} + \frac{\Delta_{IV}L}{\frac{1}{2}\rho V^2} \tag{21}$$

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Then the incremental pitching moment is

$$\frac{\Delta M}{\frac{1}{2}\rho V^2} = \frac{\Delta_I M}{\frac{1}{2}\rho V^2} + \frac{\Delta_{IV} M}{\frac{1}{2}\rho V^2} - c_r \frac{\Delta L}{\frac{1}{2}\rho V^2} \tag{22}$$

The several moments and lifts in these two equations may be taken from equations (17) to (20), using for C_I and C_{IV} therein the values determined in equations (15). Then the left sides may be converted to coefficient form by dividing equation (21) by S and $qc/2V$ and equation (22) by Sc and $qc/2V$. Multiplication by a factor of 2 accounts for both wing panels on the complete wing. The final results are, with both sides multiplied by β,

$$\Delta\left(\beta C_{L_q}\right) = -\frac{A\beta m G(m)}{n}\left\{4\frac{c_r}{c}\left[1 - \frac{\pi/2}{K'(n)}\right] + \frac{2}{3}\frac{\bar{c}}{c}\frac{A\beta}{n}\left[1 - \frac{E'(n)-n^2K'(n)}{K'(n)-E'(n)}\right]\right\} \tag{23}$$

$$\Delta\left(\beta C_{m_q}\right) = \frac{A\beta m G(m)}{n}\left\{4\frac{c_r^2}{c^2}\left[1 - \frac{\pi/2}{K'(n)}\right] + \frac{2}{3}\frac{c_r}{c}\frac{\bar{c}}{c}\frac{A\beta}{n}\left[2 - \frac{E'(n)}{K'(n)} - \frac{E'(n)-n^2K'(n)}{K'(n)-E'(n)}\right] + \frac{1}{6}\frac{\bar{c}^2}{c^2}\frac{A^2\beta^2}{n^2}\left[1 - \frac{\pi}{4}\frac{1-n^2}{K'(n)-E'(n)}\right]\right\} \tag{24}$$

Equation (24) gives the value of a correction for the damping-in-pitch derivative C_{m_q} when the trailing edge of the wing is subsonic. This correction is algebraically additive to the value of C_{m_q} computed by integration of the basic delta-wing lift distribution alone. Equation (23) gives a corresponding correction for the less-important lift-due-to-pitching derivative C_{L_q}. Both equations are subject to the Mach number-plan form limitations expressed by equations (11a) and (11b).

Equations (23) and (24) refer to a wing pitching about its apex with moments referred to the apex. The following transformations will provide values of the corrections to the derivatives (designated by primes) referred to an axis a distance h downstream of the apex:

$$\left.\begin{aligned}\Delta\left(BC_{L_q}'\right) &= \Delta\left(BC_{L_q}\right) - \frac{2h}{c}\,\Delta\left(BC_{L_\alpha}\right)\\ \Delta\left(BC_{m_q}'\right) &= \Delta\left(BC_{m_q}\right) - \frac{h}{c}\left[2\Delta\left(BC_{m_\alpha}\right)-\Delta\left(BC_{L_q}\right)\right] - \frac{2h^2}{c^2}\,\Delta\left(BC_{L_\alpha}\right)\end{aligned}\right\} \tag{25}$$

The quantities $\Delta\left(BC_{L_q}\right)$ and $\Delta\left(BC_{m_q}\right)$ on the right side are given by equations (23) and (24). The quantities $\Delta\left(BC_{L_\alpha}\right)$ and $\Delta\left(BC_{m_\alpha}\right)$ may be obtained from reference 1 (corrected in the errata sheet thereto).

Similar procedures for the treatment of the effect of subsonic trailing edges on the derivatives C_{l_β} (effective dihedral) and C_{l_r} are considered briefly in appendix C.

DISCUSSION

The circumstances under which the approximations used herein lead to acceptably small error in the calculation of the wing loading require examination. There are two points of departure from "exact" linearized potential-flow theory employing the Kutta condition at the trailing edge: (1) the basic delta-wing load distribution behind the wing is canceled only approximately by superposition in each case of a special flow, and there remains a generally small residual lift in the wake (fig. 2); (2) the mutual interaction between the tips and the trailing edge (black regions of fig. 1) is neglected.

In more elaborate treatments (reference 1 for angle of attack, reference 2 for rolling motion) the lift in the wake is canceled almost completely, and the mutual interaction between the tip and the trailing edge is also taken into account. In reference 1 two examples are presented from which the separate contributions of the several degrees of approximation may be assessed. (Similar examples in reference 2 are not so well suited to such an assessment, because of an imperfect correspondence. One of these examples is, however, hereinafter considered.) The examples refer to an untapered wing of

aspect ratio 1.72 and a tapered wing of aspect ratio 3.85 and taper ratio 0.179, both with 63° leading-edge sweepback. The Mach number is 1.5. The breakdown for lift is as follows:

Source of Lift	Untapered wing (percent)	Tapered wing (percent)
(1) Basic delta-wing loading	100.0	100.0
(2) Tip effect	-14.7	-1.6
(3) Symmetrical wake correction	-12.1	-2.7
(4) Oblique elements in wake	-.82	-.59
(5) Tip-trailing-edge mutual interference	3.0	.04
Total lift	75.4	95.1

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The "symmetrical wake correction" is cancellation flow I of this report (fig. 2). The "oblique elements in the wake" is an additional flow that, added to flow I, (almost exactly) completes the cancellation of the basic delta-wing lift behind the wing. Thus for the untapered wing the item of -0.82 percent represents the error in computed wing loading associated with the failure of flow I to cancel completely the basic delta-wing loading behind the wing. Finally, the item 3.0 percent represents the error associated with neglect of the mutual interference between the tips and the trailing edge. The errors of the present treatment therefore amount to -0.82 + 3.0 or 2.2 percent for the calculation of the lift of the untapered wing. (Note that these errors are expressed as percentages of item 1.)

The examples afford a similar breakdown for the pitching moment about the wing apex:

Source of Moment	Untapered wing (percent)	Tapered wing (percent)
(1) Basic delta-wing loading	100.0	100.0
(2) Tip effect	-23.9	-2.7
(3) Symmetrical wake correction	-17.7	-4.0
(4) Oblique elements in wake	-1.37	-.95
(5) Tip-trailing-edge mutual interference	5.3	------
Total pitching moment	62.3	92.3

The errors of the present treatment for the calculation of the pitching moment for the untapered wing therefore come to -1.37 + 5.3 or 3.9 percent and for the tapered wing to -0.95 percent (again based on item (1) = 100 percent).

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It is possible to infer, in a general way, from these examples the circumstances under which the approximations of this report, namely, the neglect of items (4) and (5), are reasonably satisfactory. The difference between cancellation flow I and the basic delta-wing lift in figure 2 corresponds to item (4). For a given section A-A across the wake, this difference decreases with the ratio QR/PS. For the evaluation of item (4) section A-A may be taken to pass through the trailing ends of the wing tips (that is, QR equals wing span b and P and S lie on the leading edges extended). The ratio QR/PS for this section is designated τ. In the untapered-wing example, $\tau = 0.629$; in the tapered-wing example, $\tau = 0.926$.

Now consider variations of wing sweep, taper, and Mach number that leave τ constant. It is expected that item (4) will change approximately in proportion to item (3) (exactly, if the trailing-edge-sweep - Mach number parameter n is unchanged). On this basis item (4) in the examples may be adjusted to equal 1 percent and item (3) scaled up accordingly. The results are

τ	Item (3) in	
	C_{L_α}	C_{m_α}
	(percent)	(percent)
0.629	-14.7	-12.9
.926	-4.6	-4.2

These four points are plotted in figure 4. A straight-line variation of τ against item (3) is not inconsistent with the points. The line drawn in is considered to represent approximately the condition that item (4) equals 1 percent. Accordingly, for values of τ below the line the neglect of item (4) will yield an error of less than 1 percent (based on basic delta-wing loading = 100 percent). The parameter τ may be calculated from the formula

$$\tau = \frac{\frac{1}{m}}{\frac{1}{m} + \frac{4\lambda}{AB(1+\lambda)}} \tag{26}$$

A corresponding criterion for the neglect of item (5), the mutual interference between tips and trailing edge, cannot be formulated with much confidence. The ratio of the area moment of the wing regions affected by this interference (black regions in fig. 1) to the area moment of the entire wing surface is presumed to be a rough guide in the estimation of moments. In the tapered-wing example both item (5) and this ratio for the area pitching moments are negligible. In the untapered-wing example this ratio for the area pitching moments is 17 percent and item (5) is 5.3 percent. For the same wing this ratio for the area rolling moments is 15.4 percent and item (5) for rolling motion is 4.3 percent. (See subsequent example, taken from reference 2.) If the correspondence can be stretched somewhat, the rule that the percent error in calculated moment associated with neglect of item (5) is about one-third the percent area-moment ratio may be used as a tentative criterion. Equations (11a) and (11b) constitute an additional limitation that states, in effect, that the black interference regions in figure 1 may not impinge on portions of the leading edge. This limitation was introduced partly for convenience in the integrations; both this and the area-moment limitation act in the same direction. Of the two, that one governs that results in the smaller area-moment ratio.

These several criterions for the suitability of the approximations of this report were inferred from examples worked out for a wing at an angle of attack. The similarity of the features of the load cancellation for rolling motion and for pitching motion (fig. 2) suggests that these criterions are of the right order for these more general motions. (Note that the appropriate axis, in each case, should be used for the area-moment criterion.) An indication that the criterion for τ is not unconservative for the case of rolling comes from the following calculation of C_{l_p} for the untapered wing of reference 1, taken (except for item (3)) from reference 2:

Source of Moment	Untapered wing (percent)
(1) Basic delta-wing loading (reference 2)	100.00
(2) Tip effect (reference 2)	-37.54
(3) Partial trailing-edge correction (equation (9) herein)	-8.06
(3a) Total trailing-edge correction (reference 2)	-8.23
(5) Tip-trailing edge mutual interference	4.26

Item (3) is the approximate trailing-edge correction computed by equation (9) of the present paper. Item (3a) is the trailing-edge correction computed in reference 2 by substantially exact cancellation

of the residual lift in the wake. Thus the difference between items (3a) and (3) corresponds to item (4) of the earlier examples. This difference is very small: only 0.17 percent of the uncorrected value of C_{l_p}, item (1). This value for item (4) is considered to have some uncertainty in view of the fact that item (3a) was obtained by numerical integration. If the point $\tau = 0.629$, item (3) = 8.06 percent, is plotted on figure 4 it falls, as it should, within the region for which item (4) is less than 1 percent.

So far the approximations in this report have been justified solely on the basis of their closeness to the values required by the "exact" linearized potential theory. For real wings, however, the action of viscosity in producing a boundary layer partly vitiates the predictions of potential theory. The boundary-layer flow for sweptback wings is such that the deviation from potential theory is expected to be appreciable in the vicinity of the wing tips and the trailing edge. Thus, the approximations that have been introduced herein are probably within the accuracy of applicability of the potential theory itself, so long as the several criterions have been met.

Lewis Flight Propulsion Laboratory,
National Advisory Committee for Aeronautics
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APPENDIX A

SYMBOLS

The following symbols are used in this report:

A	aspect ratio
b	span
C, C_I, C_{IV}	constants
C_{L_q}	coefficient of lift due to pitching $\left[\dfrac{\text{lift}}{\frac{1}{2}\rho V^2 S\left(\frac{qc}{2V}\right)}\right]$
C_{L_α}	lift-curve slope $\left(\dfrac{\text{lift}}{\frac{1}{2}\rho V^2 S\alpha}\right)$
C_{l_p}	coefficient of damping in roll $\left[\dfrac{\text{rolling moment}}{\frac{1}{2}\rho V^2 Sb\left(\frac{pb}{2V}\right)}\right]$
C_{l_r}	coefficient of rolling moment due to yawing $\left[\dfrac{\text{rolling moment}}{\frac{1}{2}\rho V^2 Sb\left(\frac{rb}{2V}\right)}\right]$
C_{l_β}	coefficient of rolling moment due to sideslip $\left(\dfrac{\text{rolling moment}}{\frac{1}{2}\rho V^2 Sb\beta}\right)$
C_{m_q}	coefficient of damping in pitch $\left[\dfrac{\text{pitching moment}}{\frac{1}{2}\rho V^2 Sc\left(\frac{qc}{2V}\right)}\right]$

C_{m_α} coefficient of pitching moment due to angle of attack $\left(\frac{\text{pitching moment}}{\frac{1}{2}\rho V^2 Sc\alpha}\right)$ 1300

C_p pressure coefficient (lift per unit area divided by $\frac{1}{2}\rho V^2 Sc\alpha$)

c mean aerodynamic chord $\left[\frac{1}{S}\int_{-b/2}^{b/2} (\text{local chord})^2\, dy\right]$

$\bar{c}$ average chord $\left[\frac{1}{b}\int_{-b/2}^{b/2} (\text{local chord})\, dy\right]$

c_r root chord

$E(\phi,k)$ incomplete elliptic integral of second kind with amplitude ϕ and modulus k

$E(k)$ complete elliptic integral of second kind with modulus k $\left[= E\left(\frac{\pi}{2},k\right)\right]$

$E'(m) = E\left(\sqrt{1-m^2}\right)$

$E'(n) = E\left(\sqrt{1-n^2}\right)$

$F(\phi,k)$ incomplete elliptic integral of first kind with amplitude ϕ and modulus k

$f(\sigma)$ function of σ defined in text

$$G(m) = \frac{1-m^2}{(1-2m^2)E'(m)+m^2K'(m)}$$

h distance of center of gravity downstream of wing apex

$I(m) = \dfrac{2(1-m^2)}{(2-m^2)E'(m)-m^2K'(m)}$

$K(k)$ complete elliptic integral of first kind with modulus $k \left[= F\left(\frac{\pi}{2},k\right)\right]$

$K'(m) = K\left(\sqrt{1-m^2}\right)$

$K'(n) = K\left(\sqrt{1-n^2}\right)$

k modulus of elliptic integral $\left(= \sqrt{1-n^2}\right)$

L lift

L' rolling moment (positive in sense of right-hand screw proceeding upstream)

M pitching moment (positive in sense of right-hand screw proceeding spanwise to right)

m β times contangent of leading-edge sweep angle measured from y-axis

n β times contangent of trailing-edge sweep angle measured from y-axis

p angular velocity of rolling

q angular velocity of pitching

r angular velocity of yawing

S wing area

u,v,w disturbance velocity components along x-, y-, and z-axes, respectively

V stream velocity

x,y,z Cartesian coordinates: x-axis parallel to free-stream direction; y-axis horizontal and toward the right, looking upstream; z-axis vertically upward

α angle of attack

β $\sqrt{M^2-1}$; also angle of sideslip

λ taper ratio (tip chord/root chord)

ρ density of air

σ = $\beta y/x$

τ = $\left(\frac{QR}{PS}\right)_{QR=b}$ (See fig. 2 and discussion in text)

ϕ amplitude of elliptic integral $\left(= \sin^{-1}\sqrt{\frac{1-\sigma^2}{1-n^2}}\right)$

Subscripts:

Δ basic delta-wing flow

I flow I

IV flow IV

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APPENDIX B

AIDS TO INTEGRATION

The integrations in this paper that involve elliptic integrals are most easily evaluated with the aid of the following elliptic function substitutions, together with the tables of integrals in references 7 and 8, and integration by parts:

$$F(\phi,k) = u$$

$$E(\phi,k) = \int_0^u dn^2 u \, du$$

$$\sqrt{1-\sigma^2} = k \, sn \, u$$

$$\sqrt{\sigma^2-n^2} = k \, cn \, u$$

$$\sigma = dn \, u$$

$$d\sigma = -k^2 \, sn \, u \, cn \, u \, du$$

where

$$\phi = \sin^{-1}\sqrt{\frac{1-\sigma^2}{1-n^2}}$$

$$k = \sqrt{1-n^2}$$

$$k' = \sqrt{1-k^2} = n$$

APPENDIX C

EXTENSION TO C_{l_β} AND C_{l_r}

The antisymmetric component of the basic delta-wing loading is qualitatively similar for rolling p, sideslip β, and yawing r. Thus the procedure developed herein for the damping-in-roll derivative C_{l_p} may be adapted without difficulty to the effective-dihedral derivative C_{l_β} and, once the correct delta-wing loading is known for yawing, to the derivative C_{l_r}. This procedure will, however, be less accurate for C_{l_β} than for C_{l_p}, because the delta-wing loading to be canceled in the wake is less well approximated by flow III. An approximation that is closer for sideslip, but still simple, is a flow specified by $u = Cy/(c_r+x)$ in the region $0 \leq |\sigma| \leq n$ and $w = 0$ in the regions $n \leq |\sigma| \leq 1$. This flow is difficult to solve for u in the $w = 0$ regions, but a flow closely similar in the right-hand $w = 0$ region may be readily solved by a procedure in reference 4. This similar flow is obtained by replacing $w = 0$ by $u = 0$ in the left-hand region $-1 \leq \sigma \leq -n$. The final solution for u in the right-hand region $n \leq \sigma \leq 1$ is to be applied for both $n \leq \sigma \leq 1$ and $-1 \leq \sigma \leq -n$.

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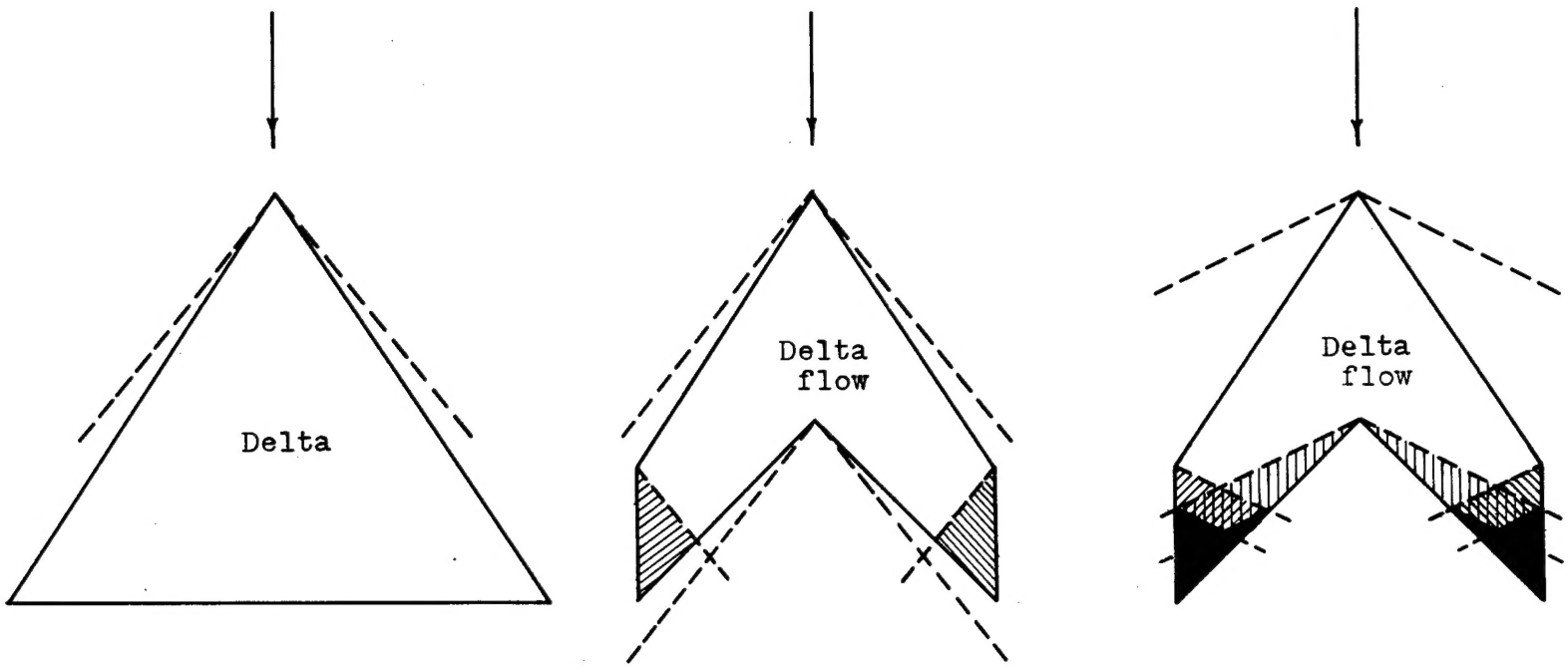

(a) Delta wing.

(b) Sweptback wing formed from delta wing.

(c) Sweptback wing formed from delta wing at lower supersonic Mach number.

Figure 1. - Related wings in supersonic flow. Hatched regions explained in text.

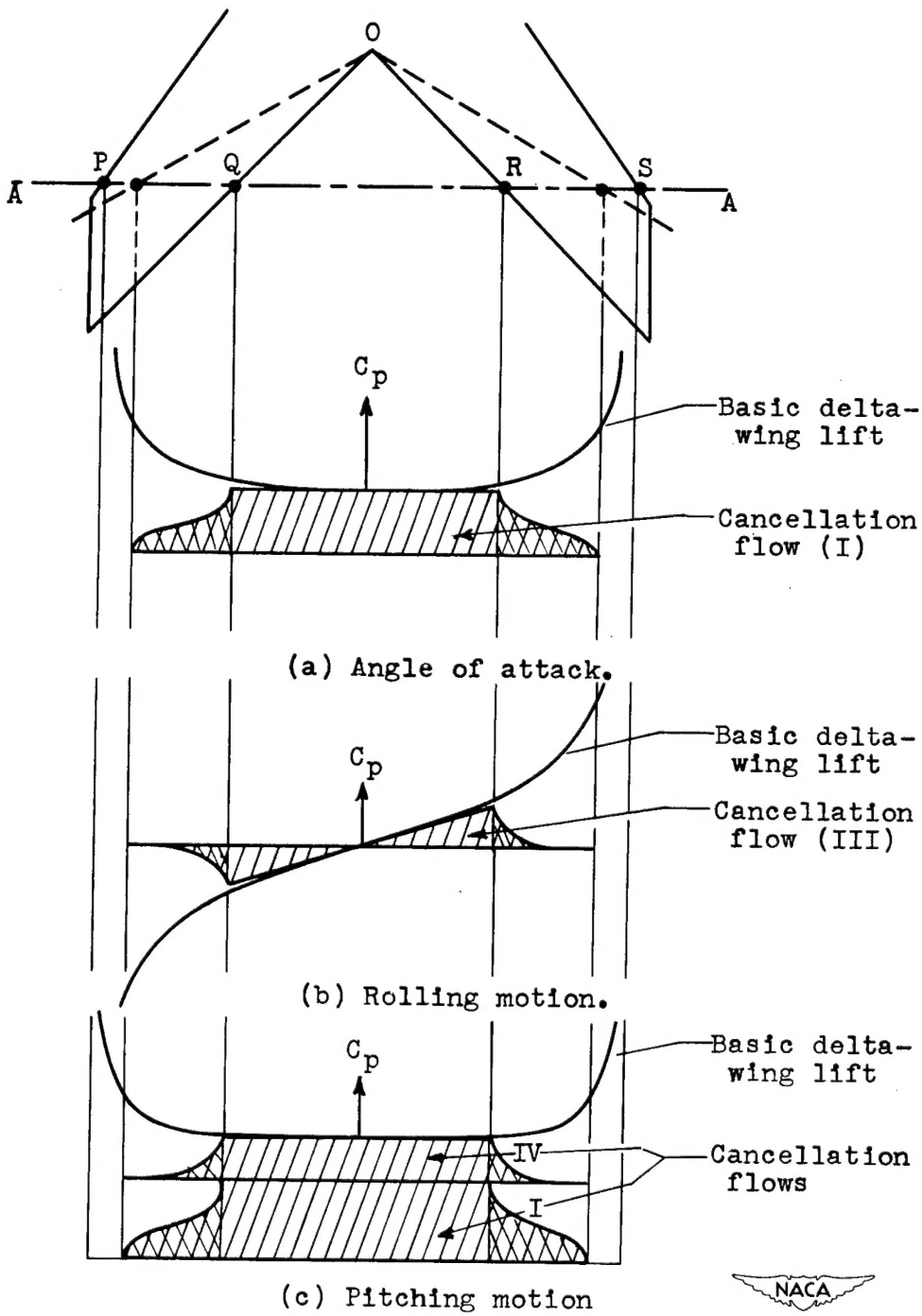

Figure 2. - Sweptback wing showing basic delta-wing load distribution along section A-A and approximate cancellation of this load behind wing by superposition of special flows. Special flows are plotted with reversed sign. Cross-hatched areas represent induced changes in loading ahead of trailing edge.

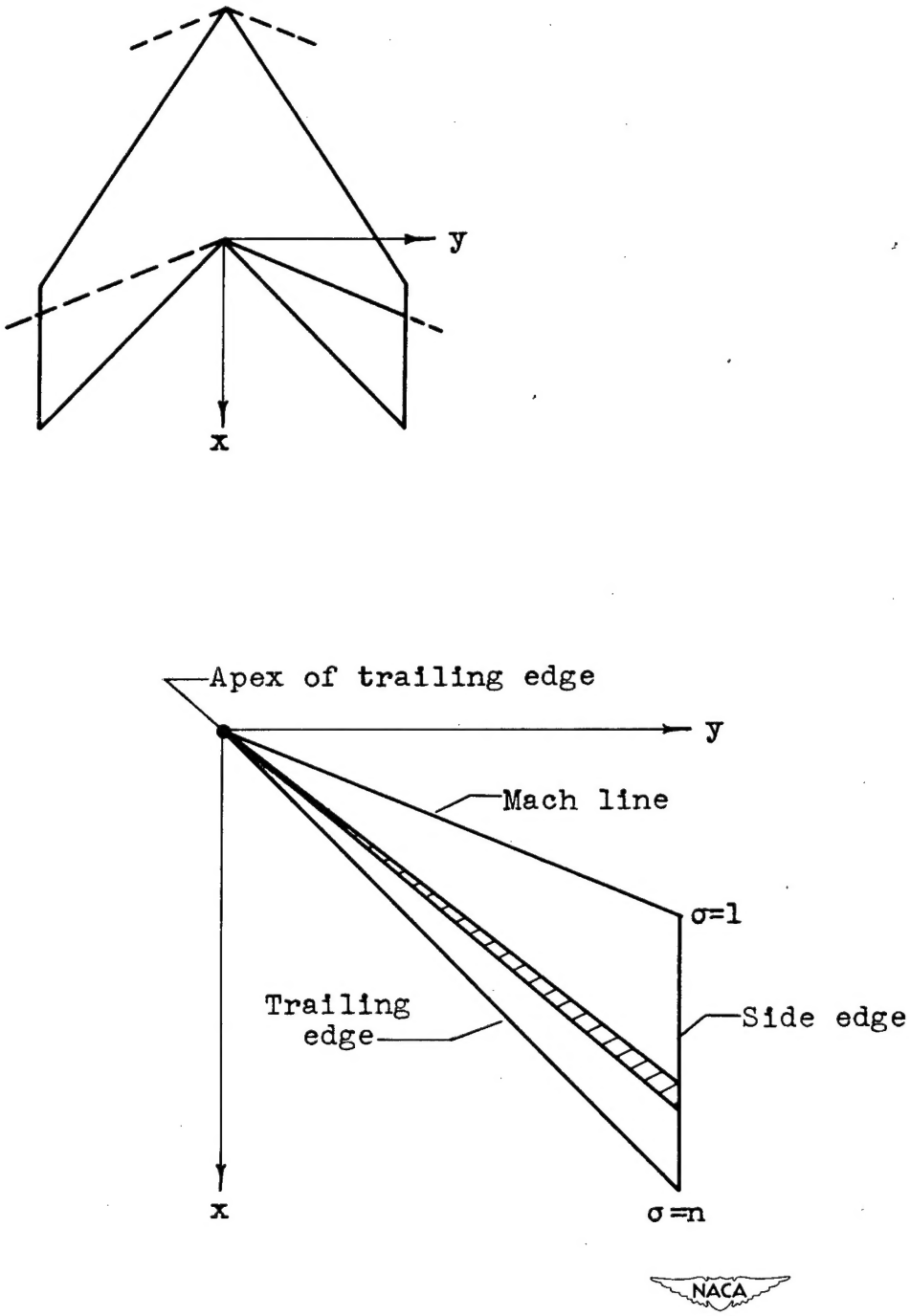

Figure 3. - Disturbed portion of trailing edge of right wing panel.

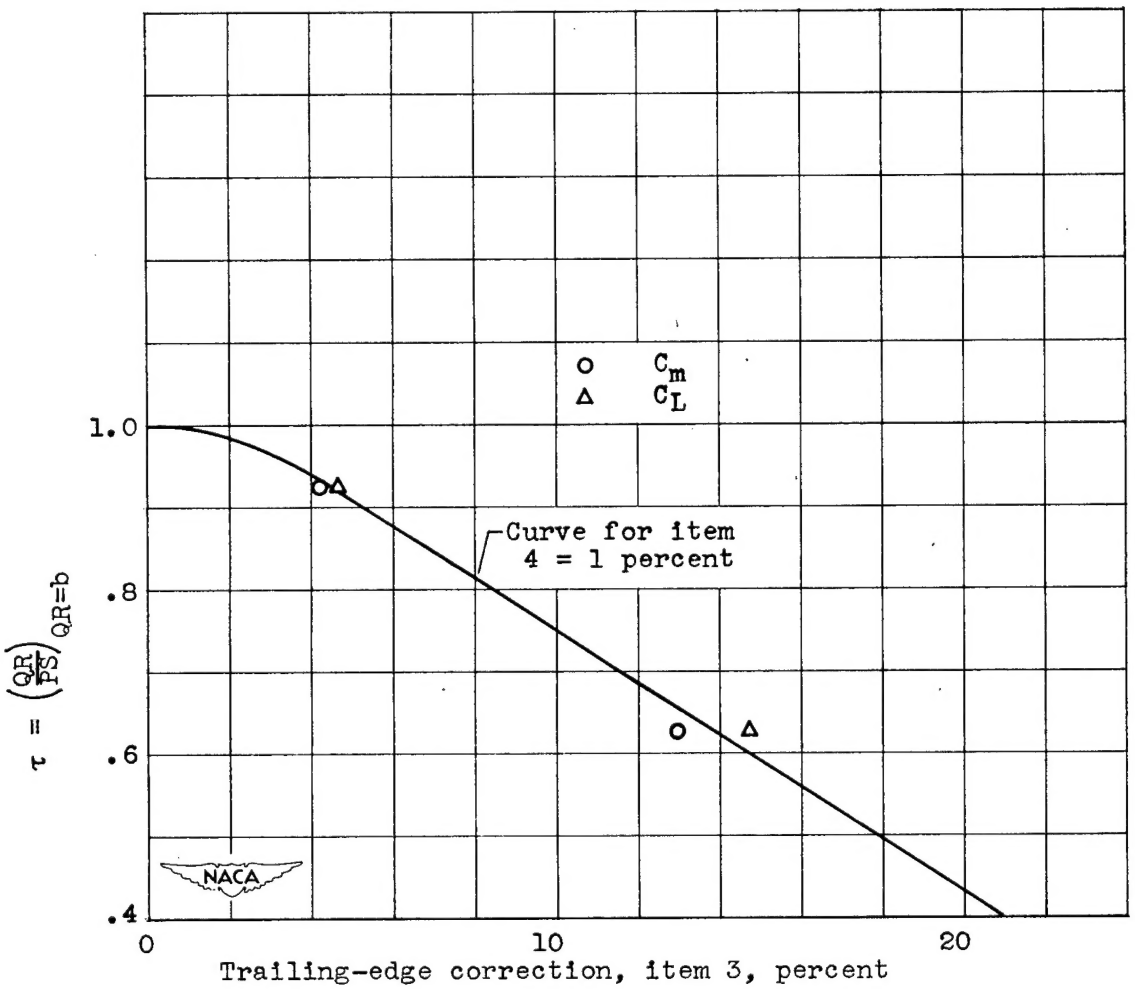

Figure 4. - Estimated variation of τ with trailing-edge correction computed by present method under condition that error in correction, exclusive of tip interaction, equals 1 percent of contribution of basic delta-wing loading.